I think this Problem can be expressed in this way :
   Do the partition function always converge ? What should we do if not ?

$$\Xi = \sum_{N=0}^{\infty} \prod_{states}^{\square} \text{Exp}[-\alpha N - \beta E]$$

Note : When N is 0, the system has only one possible state,
and E should be 0 in the state. Thus Ξ should be 1.

$$\Xi = \sum_{N=0}^{\infty} z^N Q_N (V, T)$$

$$Q_N (V, T) = \sum_{states} \text{Exp}[-\beta E]$$

When we are discussing the classical solid,

$$Q_N (V, T) = Q_1 (V, T)^N \equiv \phi (T)^N$$

Thus, $\Xi = \dfrac{1}{1 - z\phi (T)}$, when $| z\phi (T) | < 1$

First discuss for harmonic oscilators

```
ϕ[_T] = (2 Sinh[((h) (w)) / (2 (k) T)])^-3
z[_T] = Exp[μ / ((k) T)]

Plot[(2 Sinh[1 / x])^-3, {x, 0, 100}]

Plot[Exp[1 / x], {x, 0, 100}]
```

```
Plot[Exp[-1 / x] (1 - Exp[-1 / x]), {x, 10, 100}]
```

Summary

1. T → 0, T > 0
μ > 3 / 2 h (bar) ω, = > zϕ>1
μ < 3 / 2 h (bar) ω, = > zϕ<1
μ can be less than 0
2. T → +∞
zϕ → ∞
3. T → 0, T < 0
μ > 3 / 2 h (bar) ω, = > |zϕ|<1
μ < 3 / 2 h (bar) ω, = > |zϕ|>1
4. T → -∞
| zϕ | → ∞

e.g.
I checked the chemical potentials in www.job - stiftung.de. I found that generally speaking,
chemical potentials are in the range - 10 000 ~ 10 000 (kJ / mol),
(even - 10 000 ~ 1000 (kJ / mol)).That is to say,
chemical potential per particle is $\left(-1.7 * 10^{-17}, 1.7 * 10^{-17}\right)$ (kJ / mol) low tempterature :
(h (bar) ω) / (kT) is just about at the range of 0 to 1
in Debye ' s model, $\omega_D < 2 \pi * 10^{13}$ s^{-1}
k = $1.38 * 10^{-23}$ blablabla
h $((bar) \sim 10)^{-34}$
positive μ, generally | zϕ | << 1
negative μ, always | zϕ | >> 1
One thing we should pay attention to is that,
all same harmonics model is only valid at relatively high temperature (not so high as we expect) when the oscillations are almost the same.

In a non - harmonic system,
it canbe infered that the similar property at the limits in our discussion.

However, what we r really interested is How to calculate the thermal properties of of such a system when | zϕ | > 1