Statistical Physics Summary for the Final Exam

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Part 1

Microcanonical ensemble (N,V,E)

1. Equal of priori probabilities:
   Size of the phase space

$$\Omega(E) = \frac{1}{N!h^{3N}\int_{E<H<E+\Delta E} dqdp}$$

2. Observables:

$$\bar{A} = \frac{1}{\Omega}\frac{1}{N!h^{Nr}\int A(q,p)d\Omega}$$

3. Why ensembles?
4. $S = kln\Omega$
5. $$\begin{cases} dln\Omega = \beta dE + \gamma dV + \alpha dN \\ dU = TdS - PdV + \alpha N \\ lnm! = mlnm - m \end{cases}$$

Part 2

Canonical Ensemble (N,V,T).

1. $\rho = \frac{1}{Z}e^{-\beta E_s}$.

   Considering the degeneration: $\rho_l = \frac{\omega_l}{Z}e^{-\beta E_l}$

2. $\bar{A} = \frac{1}{Z}\sum_l A_l\omega_l e^{-\beta E_l} = \sum_l A_l\rho_l$

3. Tricks: $\sum_l \omega_l -> \sum_{q,p}\frac{\Delta p\Delta q}{N!h^{Nr}} -> \frac{1}{N!h^{Nr}}\int dpdq$, here N! comes from the

undistinguishable particles.

4. $$\begin{cases} P = \frac{1}{\beta}\frac{\partial lnZ}{\partial V} (\bar{Y} = -\frac{1}{\beta}\frac{\partial lnZ}{\partial y}) \\ F = -\frac{1}{\beta} lnZ \\ S = k(lnZ - \beta\frac{\partial lnZ}{\partial \beta}) \end{cases}$$

5. Microcanonical ensemble and Canonical ensemble are equivalent to each other at thermal dynamic limit.
6. Equipartition theorem. (For Classical systems.)
7. Virial theorem: $\left\langle q_i \frac{\partial H}{\partial q_i} \right\rangle = \langle q_i \dot{p}_i \rangle = kT$.
8. Heat capacity:
    a. Harmonic oscillators:
    Write down the Hamiltonian;
    Calculate the partition function Z;
    Calculate the observables according to the following formulas:

$$\begin{cases} U = -\frac{\partial lnZ}{\partial \beta} \\ C_v = \frac{\partial U}{\partial T} \\ F = -\frac{1}{\beta} lnZ \\ \mu = \frac{\partial F}{\partial N} \\ F = U - TS -> S = \text{¡-} \end{cases} ;$$

9. Heat capacity of solids:
    Write down the decoupled Hamiltonian -> Cal. Z -> Cal. U.
    Then the road diverges!
    a. Einstein Model: $\omega_i = \omega$
    b. Debye Model: (or continuous model) Assume the frequency distribution is: $g(\omega)d\omega = B\omega^2 d\omega$, with ω_D as the cutoff
    ($\int_o^{\omega_D} g(\omega)d\omega = 3N$).

Part 3

Quantum statistics.

1. DM: $\hat{\rho} = \frac{1}{N}\sum_{k=1}^{N} |\Psi_k\rangle\langle\Psi_k|$, in which $a_k^n = \langle \phi_n | \Psi_k \rangle$.
2. Von Neumann Eq: $i\hbar\dot{\rho} = \left[\hat{H}, \hat{\rho}\right]$. $\dot{\rho} = 0$ when the system is under equilibrium.
3. Microcanonical ensemble: $\rho_n = \begin{cases} \frac{1}{\Gamma} \\ 0 \end{cases}$

   Canonical ensemble: $Z = e^{\beta\mu}$. $\rho = \frac{e^{-\beta H}}{\{tre^{-\beta H}\}}$. $Q(\mu, V, T) = \sum_{N} =0^{\infty} z^N Q_N(\beta)$

   $\langle \hat{A} \rangle_c \equiv tr\{\hat{A}\hat{\rho}\}$.

4. Details:
   a. Diagonalise Hamiltonian.
   b. Cal. DM: $\hat{\rho}$
   c. Cal. Observables: $\langle \hat{A} \rangle_c$
5. Thermal length: $\lambda \equiv \{\frac{h^2}{2\pi mkT}\}^{1/2}$.

Part 4

1. Universality: deep rooted in the correlation length.
2. Critical exponent.
3. Ising Model:

a. Landau:

$$\left\{\begin{array}{l} \mu(T,m)=\mu_0(T)+a(T)m^2+\frac{1}{2}c(T)m^4+etc. \\ a=a_1t, c=c_0 \\ Equilibrium\frac{\partial\mu}{[partialm}=0,\frac{\partial^2\mu}{[partialm^2}>0 \\ => m \\ \mu(T,m)=-Bm+\mu_0(T)+a(T)m^2+\frac{1}{2}c(T)m^4+etc. \\ => \chi=\frac{\partial m}{\partial B} \end{array}\right.$$

b. MFT:

(1) Hamiltonian: $H=-J\sum_{<i,j>}\sigma_i\sigma_j-\mu B\sum_i\sigma_i$

➔ $H=\frac{1}{2}JN\bar{\sigma}^2z-Jz\bar{\sigma}\sum_i\sigma_i-\mu B\sum_i\sigma_i=\frac{1}{/}2JNz\bar{\sigma}^2-\mu(B+\frac{J}{z}\mu\bar{\sigma})\sum_i\sigma_i$

(2) Partition func.: $Z_N=Z_i^N=\{e^{-\beta\mu B_{eff}}+e^{\beta\mu B_{eff}}\}^N$

(3) Constrains: $\bar{\sigma}=\frac{1}{\beta}\frac{\partial}{\partial H}lnZ$

(4) Observables:

T_c, when $B=0->\bar{\sigma}=\tanh\left(\frac{zJ}{kT}\bar{\sigma}\right)\ ->\ Tc=\frac{zJ}{k}$

Near the critical zone, $\bar{\sigma}\ <-\ \tanh\left(\beta\mu B_{eff}\right)$

$C(B=0)=\frac{dU}{dT}$, $U=\langle H\rangle=-\frac{1}{2}Jz\bar{\sigma}^2N$

$\chi(B,T)=\frac{\partial\bar{M}}{\partial B}_T=N\mu\left(\frac{\partial\bar{\sigma}}{\partial B}\right)_T$

c. Transfer Matrix method.

d. Recursion method. $Z_{N+1}=f(Z_N)$

e. Renormalization method.

4. Order parameter.